Convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet

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Abstract *Presents a numerical solution of the two-dimensional laminar boundary layer problem on free and forced convection of an incompressible visco-elastic fluid immersed in a porous medium over a stretching sheet. Here, the driving force for the flow is provided by an impermeable sheet stretched with a velocity proportional to the distance from a slit and buoyancy effects due to both temperature and concentration gradients. The resultant governing boundary layer equations are highly non-linear and coupled form of partial differential equations, and they have been solved by employing a numerical shooting technique with fourth order Runge-Kutta integration scheme. Numerical computations are carried out for the non-dimensional physical parameters. The results are analyzed for the effect of different physical parameters like visco-elasticity, permeability of the porous medium, Grashof number, Schmidt number and Prandtl number on the flow, heat and mass transfer characteristics. One of the several important observations is that the combined effect of thermal diffusion and diffusion of species is to increase the horizontal velocity profile and to decrease the temperature and concentration profiles in the boundary layer flow field.*

Nomenclature

g	= acceleration due to gravity
ν	= kinematic viscosity
k_o	= visco-elastic parameter
k'	= permeability coefficient of porous medium
$\beta^* \beta^{**}$	= volumetric thermal coefficient and concentration coefficient respectively
A,B,E	= constants in equation (5)
l	= characteristic length
k_1, k_2	= visco-elastic parameter and porosity parameter respectively
G_r, G_c	= Grashof number and modified Grashof number respectively
Pr,Sc	= Prandtl number and Schmidt number respectively
τ_w	= local skin friction parameter
N_u, S_h	= Nusselt number and Sherwood number respectively
α,β,γ	= correct initial values of $f_3(0), \theta_k(0), \phi_2(0)$ respectively
$\alpha_n,\beta_n,\gamma_n$	= n^{th} iterative values of α, β, γ respectively
f, θ, ϕ	= dimensionless values of velocity, temperature and concentration respectively.

Introduction

In recent years, a great deal of interest has been generated in the area of two-dimensional boundary layer flow over a continuous moving solid surface, in view of its numerous and wide-ranging applications in various fields like aerodynamic extrusion of polymer sheets, continuous stretching, rolling and manufacturing of plastic films and artificial fibers. Sakiadis (1961) was the first amongst others to study such problem by considering the boundary layer viscous flow over a continuous solid surface moving with constant velocity. This work is followed by the pioneering work of Crane (1970), in which the flow is caused by an elastic sheet moving in its own plane with a velocity varying linearly with the distance from a fixed point. There are several extensions to this problem, which include consideration of more general stretching velocity and the study of heat transfer (Chen and Char, 1988; Dutta *et al.*, 1985; Gupta and Gupta, 1977).

Since most of the fields considered in these applications are of non-Newtonian nature, this study has been channelised to the field of non-Newtonian fluids obeying their constitutive stress-strain relations. In certain polymer processing applications, such as 5.4 percent solution of polyisobutylene in cetane and 0.83 percent solution of ammonium alginate in water (Acrivos, 1961), the visco-elastic fluid flow occurs over a stretching sheet. In view of this application, Rajagopal *et al.* (1984) have studied the flow behaviour of a visco-elastic fluid over a stretching sheet and gave an approximate solution for the flow. Dandapat and Gupta (1989) have discussed the flow of an incompressible second order fluid over a stretching sheet and obtained an analytical solution of the non-linear constitutive equation. Rollins and Vajravelu (1991) have extended the above work to the cases when:

- the boundary sheet is maintained with prescribed surface temperature (PST); and
- the boundary sheet is maintained with prescribed heat flux (PHF).

Lawrence and Rao (1992) have discussed the physically realistic solution among the two closed-form solutions obtained from the momentum equation. Rao (1996) has studied the flow of a second-grade fluid and showed the uniqueness of such flow over a stretching sheet. Siddappa and Abel (1985) have analysed the flow of a visco-elastic fluid obeying Walters' model past a stretching sheet. The same study is further channelised to the flow through porous medium by Gupta and Sridhar (1985) and Abel and Veena (1998) without considering the heat and mass transfer phenomena. However, the heat and mass transfer phenomena in a porous medium find their applications in various engineering disciplines such as geothermal fields, soil pollution and nuclear waste disposal. The study of convection in a visco-elastic fluid flow through a porous layer has application in the production of heavy crude oils (Rudraiah *et al.* 1989). Rudraiah *et al.* (1989) have investigated visco-elastic fluid flow of the type Oldroyd fluid through a porous layer heated from below. Also, in some industrial transport processes, the driving force for the flow is provided by the combination of thermal and chemical species diffusion effects. Such situations arise in applications such as the curing of a plastic and the manufacture of pulp-insulated cables. Hence, the study of the above intricate problem, taking into account the combined buoyancy effects of both thermal diffusion and diffusion of chemical species, is of vital significance. The available literature on heat and mass transfer through porous medium (Bestman, 1989, 1990; Vajravelu, 1994) reveals that such study is not being carried out into the flow of short memory fluid of the type Walters' liquid B.

Recently, Prasad *et al.* (2000) have investigated the problem of visco-elastic fluid (Walters' liquid B) flow through porous medium without mass transfer.

Hence, in the present paper, we make an attempt to investigate the problem of convective heat and mass transfer of Walters' liquid B embedded in a porous medium over a stretching sheet. The presence of combined buoyancy effects leads to the momentum, heat and mass transfer equations in the coupled form of highly non-linear partial differential equations. To deal with the coupling and non-linearity, a numerical shooting technique for three unknown initial conditions with Runge-Kutta fourth order integration scheme has been developed. The results are analysed for various values of non-dimensional parameters on heat and mass transfer characteristics.

Mathematical formulation

Consider the flow of a visco-elastic fluid (Walters' liquid B) through a porous medium of permeability k' over a semi-infinite stretching sheet coinciding with the plane $y = 0$. The flow is generated, due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x-axis. Keeping the origin fixed, the sheet is then stretched with a speed varying linearly with the distance from the slit. Buoyancy effects due to both temperature and concentration gradients and stretching of the wall provide the driving force for the flow. Under these assumptions and neglecting Soret and Dufour effects, the basic boundary layer equations governing the flow, heat and mass transfer, in usual notations, are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} - \frac{\upsilon}{k'}u + g\beta^*(T - T_\infty) + g\beta^{**}(C - C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

Where u, v are velocity components, T and C are, respectively, the temperature and concentration of chemical species in the fluid, g is the acceleration due to gravity, υ is the kinematic viscosity, k_0 is the non-Newtonian visco-elastic parameter, k' is the permeability coefficient of porous medium, β^* is the volumetric coefficient of thermal expansion and β^{**} is the volumetric concentration coefficient. Other quantities have their usual meanings.

The boundary conditions governing the flow are:

$$u=bx, \nu=0, C=C_w+A(x/l), T=T_w=T_\infty+B(x/l)(PST\ Case)$$
$$-kT_y=q_w=E(x/l)(PHF\ Case)\ at\ \ y=0 \quad (5a)$$

$$u \to 0,\ \ u_y \to 0,\ \ T \to T_\infty\ \ and\ \ C \to C_\infty\ as\ \ y \to \infty \quad (5b)$$

To take into account the effect of stretching of the boundary sheet, and the effects due to temperature and concentration gradients, we prescribe the wall boundary conditions in the form of (5a). In order to study the heat transfer analysis we consider two general cases of non-isothermal temperature boundary conditions, namely:

(1) boundary with prescribed power law surface temperature (PST); and

(2) boundary with prescribed power law heat flux (PHF).

The faraway boundary conditions are taken in the form (5b), as free stream velocity is assumed to be zero, T_w and C_w are being free stream temperature and free concentration respectively. Here $b > 0$ is known as stretching rate. The subscript y denotes the differentiation *w.r.t.* y. Now, we introduce the following dimensionless variables:

$$u = bx\, f_\eta\,(\eta),\ \ v = -\sqrt{b\upsilon}f(\eta)\ ,\quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$
$$where\ \ T_w - T_\infty = B\left(\frac{x}{l}\right) \qquad in\ \ PST\ Case\ \ ,\ C_w - C_\infty\ = A\left(\frac{x}{l}\right)$$
$$= \frac{E}{k}\sqrt{\frac{\upsilon}{b}}\ \left(\frac{x}{l}\right) \qquad in\ \ PHF\ Case \quad and\ \eta = \sqrt{\frac{b}{\upsilon}}y \quad (6)$$

With these changes of variables equation (1) is identically satisfied and equations (2)-(4) are transformed to

$$f_\eta^2 - f\ f_{\eta\eta} = f_{\eta\eta\eta} - k_1\left\{2\,f_\eta\ f_{\eta\eta\eta} - f\ \ f_{\eta\eta\eta\eta} - \ f_{\eta\eta}^2\right\} - k_2\,f_\eta + G_r\,\theta + G_c\,\phi \quad (7)$$

$$\theta_{\eta\eta} + \Pr\ \{\ f\ \theta_\eta\ -\ \ \theta\ f_\eta\}\ \ = 0 \quad (8)$$

$$\phi_{\eta\eta} + Sc\ \left\{\ f\,\phi_\eta\ -\ \ f_\eta\ \ \phi\ \ \right\} = 0 \quad (9)$$

The corresponding boundary conditions take the form:

- *PST case*:

$$\begin{array}{ll} f\ =\ 0,\quad f_\eta = 1,\quad \theta = 1,\quad \phi = 1 & at\ \ \eta\ = 0 \\ f_\eta\ =\ f_{\eta\eta} =\ \theta\ =\ \phi =\ 0 & as\ \eta \to \infty. \end{array} \quad (10)$$

- *PHF case*:

$$\begin{array}{lllll} f = 0, & f_\eta = 1, & \theta_\eta = -1, & \phi = 1 & at\ \eta = 0 \\ f_\eta = 0, & f_{\eta\eta} = 0, & \theta = 0, & \phi = 0 & as\ \eta \to \infty \end{array} \quad (11)$$

Where subscript η denotes the differentiation with respect to η. k_1, k_2 are the visco-elastic and porosity parameters, G_r and G_c are the free convection parameters, and Pr, Sc denote Prandtl number and Schmidt number respectively. These dimensionless physical parameters are defined as:

$$\begin{array}{l} k_1 = \frac{k_0 b}{\upsilon}, k_2 = \frac{\upsilon}{k' b},\ G_r \frac{g\beta^*(T_w - T_\infty)}{b^2 x},\ G_c \frac{g\beta^{**}(C_w - C_\infty)}{b^2 x}, \\ \Pr = \frac{\mu c_p}{k} \text{ and } Sc \frac{\bar{\upsilon}}{D} \end{array} \quad (12)$$

Where expressions for $T_w - T_\infty$ and $C_w - C_\infty$ are given in equation (6). The important physical quantities of our interest are the local skin friction "τ_w," Nusselt number "N_u," and Sherwood number "S_h" and they are defined in the sequel:

$$\tau_w = \frac{\tau^*}{\mu b x \sqrt{\frac{b}{\upsilon}}} = -f_{\eta\eta}(0), \quad \text{where} \quad \tau^* = -\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (13)$$

$$N_u = -\frac{h}{t_w - T_\infty} T_y = \begin{cases} \theta_\eta(0) & in\ PST\ Case \\ \frac{1}{\theta(0)} & in\ PHF\ Case \end{cases} \quad (14)$$

$$S_h = -\frac{h}{C_w - C_\infty} C_y = \phi_\eta(0). \quad (15)$$

Numerical solution

Equations (7), (8) and (9) are highly non-linear, coupled, ordinary differential equations. In order to solve them numerically, we adopt most efficient numerical shooting technique with fourth order Runge-Kutta integration scheme. Selection of the appropriate finite values of η_∞ is most important aspect in this method. We select η_∞ following the procedure outlined in the work of Abel *et al.* (2000). For different sets of physical parameters the appropriate values of η_∞ are different.

The equations (7), (8) and (9) are solved numerically by following the principle of superposition (Na, 1979). Here, the coupled boundary value problem of fourth order in f, second orders in θ and ϕ, has been reduced to a system of eight simultaneous coupled ordinary differential equations by assuming $f = f_1,\ f_\eta = f_2,\ f_{\eta\eta} = f_3,\ f_{\eta\eta\eta} = f_4, \theta = \theta_1, \theta_\eta = \theta_2, \phi = \phi_1,$ and $\phi_\eta = \phi_2$. In order to solve this resultant system we need to have eight initial conditions, whilst, we have only two initial conditions ($f_1(0)$ and $f_2(0)$) on f, two initial conditions one each on θ and ϕ ($\theta_k(0)$, $\theta_1(0)$, k = 1 in PST case and k = 2 in PHF case).The third initial condition on f ($f_4(0)$) is obtained in terms of physical parameters by applying the initial conditions of (10) and (11) in the equation (7) (Lawrence and Rao, 1995). Since $f_3(0)$, $\theta_k(0)$ and $\phi_2(0)$ (k = 1 in PHF case, k = 2 in PST case) are not prescribed, we start with the initial approximations as $f_3(0) = \alpha_0, \theta_k(0) = \beta_0$, and $\phi_2(0) = \gamma_0$. Let α, β and γ be the correct initial values of $f_3(0)$, $\theta_k(0)$ and $\phi_2(0)$. Now we integrate the resultant system of eight ordinary differential equations using standard fourth order Runge-Kutta method and denote the values of f_3,θ_k and ϕ_1 at $\eta = \eta_\infty$ by $f_3(\alpha_0, \beta_0, \gamma_0, \eta_\infty)$, $\theta_k(\alpha_0, \beta_0, \gamma_0, \eta_\infty)$ and $\phi_1(\alpha_0, \beta_0, \gamma_0, \eta_\infty)$ respectively. Since f_3, θ_k and ϕ_1 at $\eta = \eta_\infty$ are clearly functions of α, β and γ, they are expanded in Taylor series around $\alpha - \alpha_0, \beta - \beta_0$ and $\gamma - \gamma_0$ respectively, retaining only the linear terms. We use the difference quotients for the derivatives appeared in these Taylor series expansions. Now, after solving the system of Taylor series expansions for $\delta\alpha_0 = \alpha - \alpha_0, \beta\delta_0 = \beta - \beta_0$ and $\delta\gamma_0 = \gamma - \gamma_0$ we obtain the new estimates $\alpha_1 = \alpha_0 + \delta\alpha_0, \beta_1 = \beta_0 + \delta\beta_0$ and $\gamma_1 = \gamma_0 + \delta\gamma_0$. The entire process is repeated starting with $f_1(0)$, $f_2(0)$, α_1, $f_4(0)$, β_1, γ_1 as initial conditions. Iteration of the whole outlined procedure is continued with the latest estimates of α, β and γ until the computed values at large distances coincide with the values of prescribed boundary conditions. Finally we obtain $\alpha_n = \alpha_{n-1} + \delta\alpha_{n-1}, \beta_n = \beta_{n-1} + \delta\beta_{n-1}$, $\gamma_n = \gamma_{n-1} + \delta\gamma_{n-1}$, for n = 1, 2, 3, ... as the desired most approximate initial values $f_3(0)$, $\theta_k(0)$ and $\phi_2(0)$. With this, now all the eight initial conditions become known and so we solve the resultant system of simultaneous eight equations by fourth order Runge-Kutta integration scheme and get the profiles of f_1, f_2, f_3, f_4, θ_1, θ_2, ϕ_1 and ϕ_2 for a particular set of physical parameters. The method described above is the generalization of the method outlined by Conte and de Boor (1986) to the case of three unknown initial conditions. This is analogous to the modified Newton's method of finding roots of equations in several variables.

Results and discussion

The numerical computations have been carried out for various values of visco-elastic parameter (k_1), porosity parameter (k_2), Grashof number (G_r), modified Grashof number (G_c), Prandtl number (Pr) and Schmidt number (Sc) using numerical scheme discussed in the previous section. In order to illustrate the results graphically, the numerical values are plotted in Figures 1-8. These figures depict the horizontal velocity, temperature and concentration profiles for both PST and PHF cases. Values of local skin friction ($f_{\eta\eta}(0)$), Nusselt number (N_u) and Sherwood number (S_h) in PST case are recorded in Table I.

Figures 1-3 are the graphical representation of horizontal velocity profiles $f_\eta(\eta)$ for different values of k_1, k_2, G_r and G_c. Figure 1 provides the information that the increase of visco-elastic parameter and permeability parameter leads to the decrease of the horizontal velocity profile, in the absence of free convection parameters G_r and G_c. This is because of the fact that the introduction of tensile

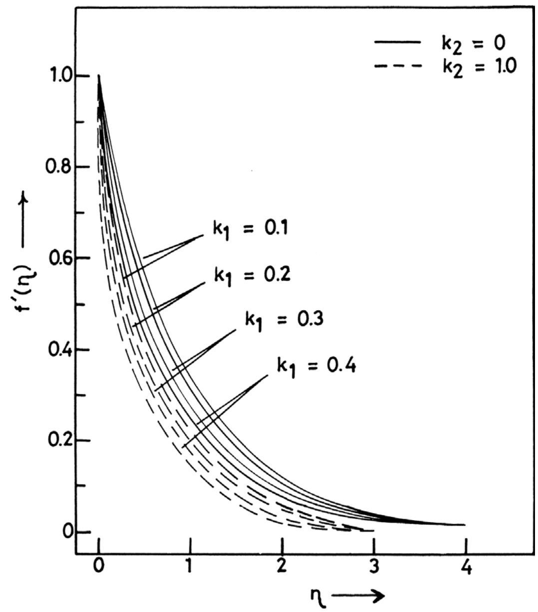

Figure 1.
Horizontal velocity profiles, f′ (n) vs. η for different values of k_1 and k_2 when $G_r = G_c = 0$

stress due to visco-elasticity causes transverse contraction of the boundary layer and the increase of porosity parameter k_2 leads to the enhanced deceleration of the flow and hence, the velocity decreases. In Figures 2(a) and (b) horizontal velocity profiles are shown for different values of Grashof number (G_r) and modified Grashof number (G_c). Physically $G_r > 0$ means heating of the fluid or

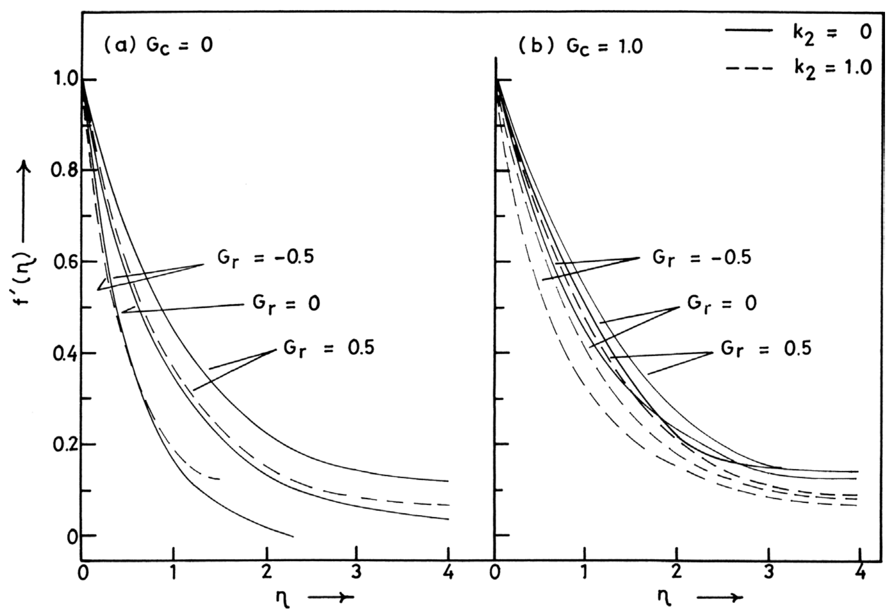

Figure 2.
Horizontal velocity profiles, f′ (n) vs. η for different values of Grashof number (a) $G_c = 0$ and (b) $G_c = 1.0$ in PST case

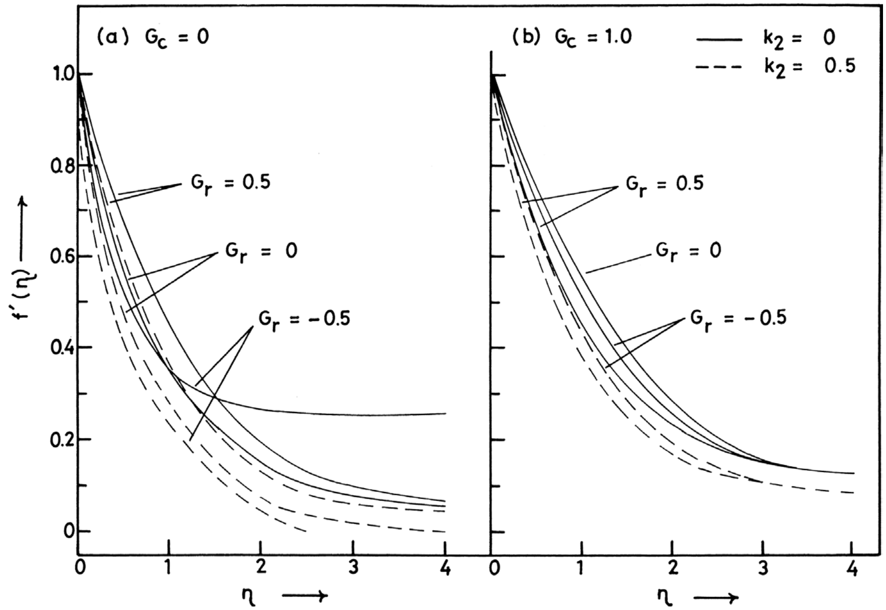

Figure 3.
Horizontal velocity profiles, f′ (η), vs. η for different values of Grashof number when (a) $G_c = 0$ and (b) $G_c = 1.0$ in PHF case

cooling of the boundary surface, $G_r < 0$ means cooling of the fluid or heating of the boundary surface and $G_r = 0$ corresponds to the absence of free convection current. This figure demonstrates that the increase of Grashof number leads to the increase of horizontal velocity profile in the absence of species diffusion (G_c = 0). This phenomenon is even true in the presence of species diffusion ($G_c = 1.0$). Increase of Grashof number (G_r) means increase of temperature gradients (T_w-T_∞), which leads to the enhancement of horizontal velocity profile due to enhanced convection. Comparison of Figures 2(a) and (b) reveals that the introduction of chemical species diffusion ($G_c \neq 0$) leads to the increase of horizontal velocity profile in both the cases of heating and cooling of the fluid. This observation is true even in the presence of porosity parameter but with reduced magnitude. This is because of the fact that the chemical species diffusion takes place due to concentration gradients, which accelerates the movement of the flow. Figures 3(a) and (b) depict the horizontal velocity profiles when the surface heat flux is prescribed (PHF) with power law profile. The effects of all the physical parameters are noticed to be qualitatively similar but quantitatively in reduced magnitude.

In Figures 4-6, temperature profiles are plotted for the same quantitative values of the physical parameters as those considered in Figures 1-3. It is noticed from these figures that the temperature distribution is unchanged (unit value) at the wall with the change of physical parameters in PST case (Figures 4 and 5). However, it changes with the change of physical parameters when the wall is maintained with prescribed wall heat flux with power law profile (Figure 6). The non-dimensional temperature distribution asymptotically reduces to zero in the free stream in both the cases of PST and PHF in

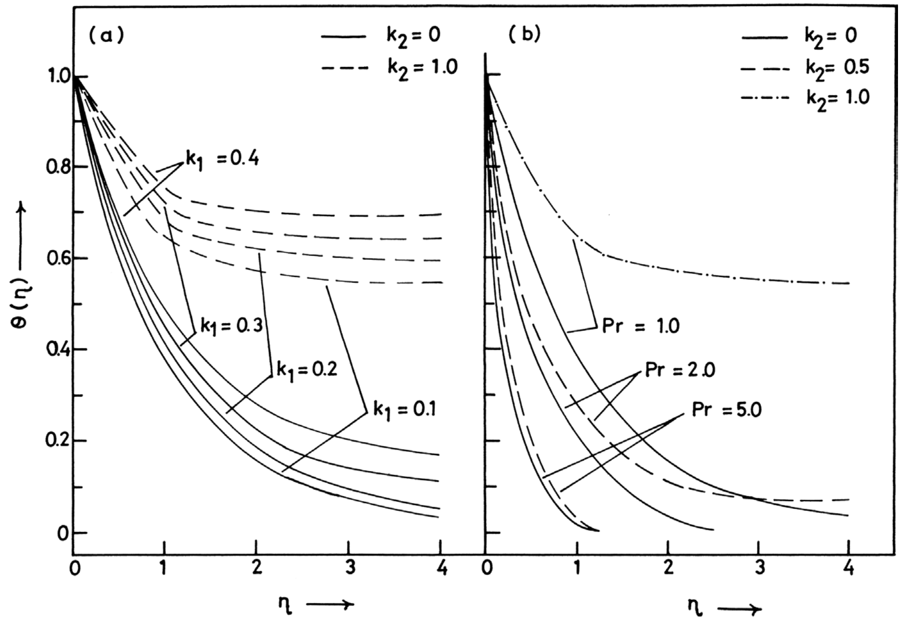

Figure 4. Temperature profile, θ (η), vs. η for different values of (a) k_1 and k_2 when $G_r = G_c = 0$, and (b) Prandtl number when $G_r = G_c = 0$ and $k_1 = 0.1$ in PST case

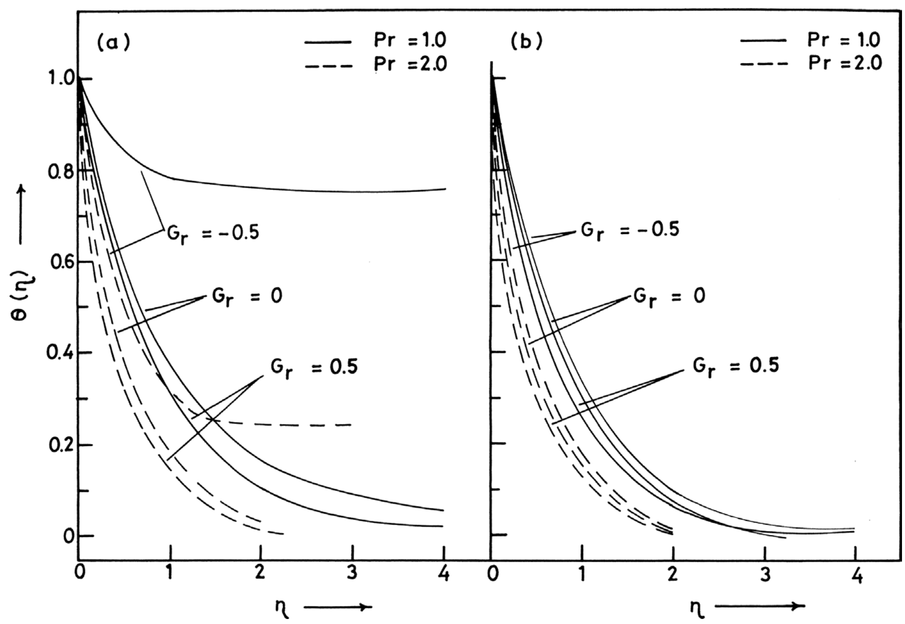

Figure 5. Temperature profile, θ (η), vs. η for different values of Grashof number (a) $G_c = 0$ and (b) $G_c = 1.0$ in PST case

conformity with the assumed faraway boundary conditions. Figure 4(a) is plotted for the effects of visco-elastic parameter and porosity parameter on temperature distribution. It is observed that the increase of visco-elastic parameter leads to the increase of temperature profile and this behaviour is

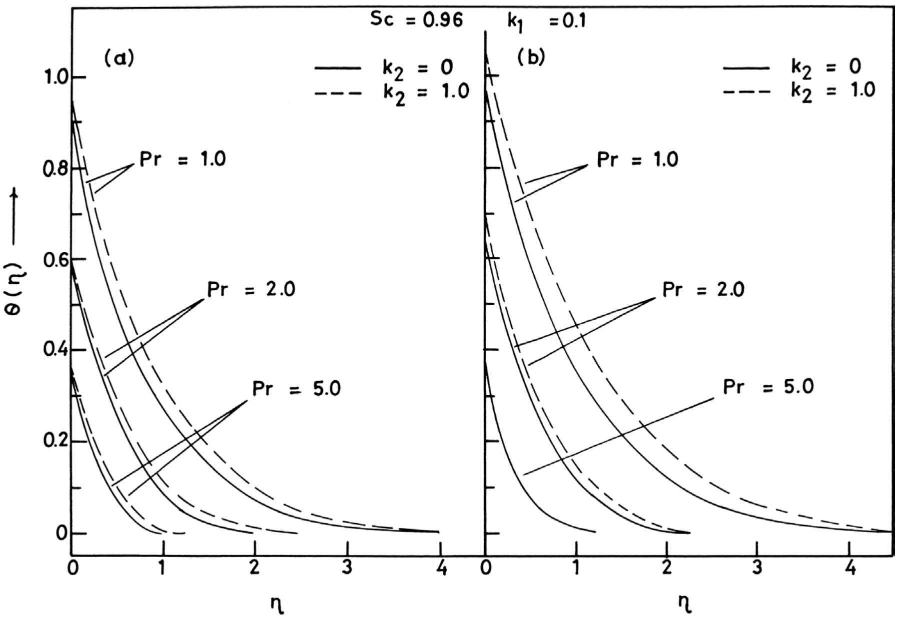

Figure 6.
Temperature profile, $\theta(\eta)$, vs. η for different values of Prandtl number when (a) $G_r = G_c = 0.5$ and (b) $G_r = G_c = 0$ in PHF case

even true in the presence of porous medium. This is consistent with the fact that the thickening of thermal boundary layer occurs due to the increase of non-Newtonian visco-elastic normal stress. Figure 4(b) reveals the effect of Prandtl number. It is seen that the increase of values of Prandtl number decreases the temperature profile. Physically, it means that the thermal boundary layer thickness decreases with the increase of the values of Prandtl number. Figures 5(a) and (b) represent the effect of modified Grashof number (G_c), that is, the effect of species diffusion in PST case. The effect of increasing the values of modified Grashof number (G_c) is to decrease the temperature profile $\theta(\eta)$ for $G_r < 0$. The effect of Prandtl number (Pr), porosity parameter (k_2), Grashof number (G_r) and modified Grashof number (G_c) on temperature profiles in PHF case is shown in Figure 6. The combined effect of increasing the values of Prandtl number, Grashof number and modified Grashof number is to reduce the temperature profile significantly in the flow field and more significantly on the wall.

The dimensionless concentration profiles are drawn in Figures 7 and 8. Figures 7(a) and (b) show the dependence of concentration profile on Schmidt number, visco-elastic parameter and porosity parameter. From Figure 7(a) we observe that the visco-elastic and porosity parameters increase the concentration distribution and it is unity at the wall. The concentration profile approaches to a constant value as the distance increases. The effect of Schmidt number (Sc) on the concentration distribution is to decrease the concentration distribution in the boundary layer (Figure 7(b)). This is due to the thinning of concentration boundary layer with the introduction of chemical species diffusion. The behaviour of Grashof number and modified Grashof number is

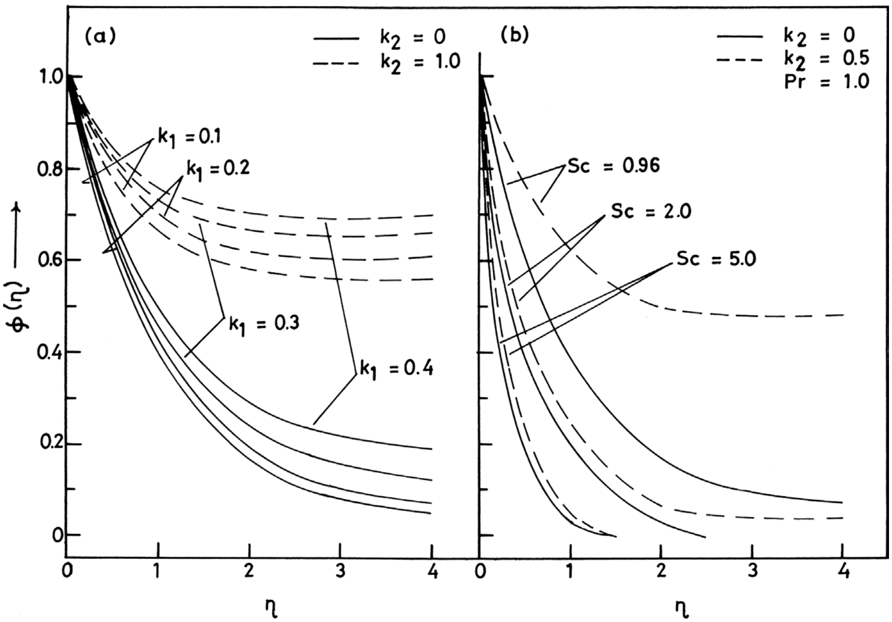

Figure 7. Concentration profile, ϕ (η), vs. η for different values of (a) k_1 and k_2 when $G_r = G_c = 0$ and (b) of Schmidt number when $G_r = G_c = 0$ and $k_1 = 0.1$

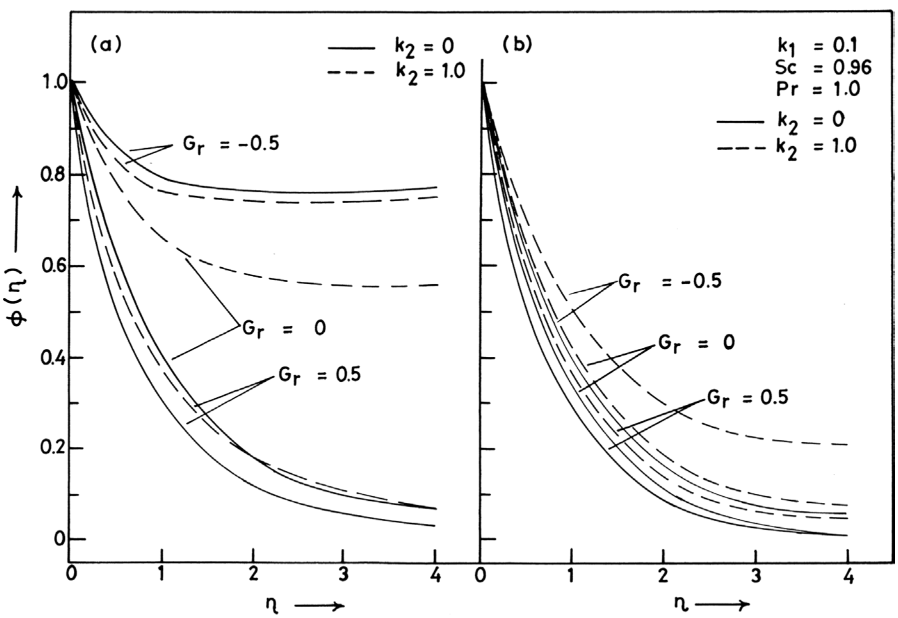

Figure 8. Concentration profile, ϕ (η), vs. η for different values of Grashof number when (a) $G_c = 0$ and (b) $G_c = 1.0$ in PST case

graphically represented in Figure 8. It is noticed that the effect of Grashof number (G_r) is to decrease the concentration distribution as the concentration species dispersed away largely due to temperature gradient. Comparison of Figures 8a and b reveals that the effect of modified Grashof number (G_c) is to

Table I. Values of skin friction parameter $f_{\eta\eta}(0)$, Nusselt number $\theta_\eta(0)$, Sherwood number $\phi_\eta(0)$ for different values of k_1 and k_2 in PST case

K_1	P_r	S_c	G_r	G_c	Skin friction k_2 (porosity) 0.0	0.5	Nusselt number k_2 (porosity) 0.0	0.5	Sherwood number k_2 (porosity) 0.0	0.5
0.1	1.0	0.96	–0.5	0.0	–1.45259	–1.59789	–0.53805	–0.55453	0.51718	–0.5334
				1.0	–0.78097	–1.04765	–1.05634	–0.98609	–1.02949	–0.9578
			0.0	0.0	–1.066	–1.3182	–0.97420	–0.77920	0.94568	–0.7482
				1.0	–0.54301	–0.81241	–1.09497	–1.0353	–1.06820	–1.0076
			0.5	0.0	–0.78945	–1.05452	–1.05113	–0.97858	–1.02435	–0.9500
				1.0	–0.30170	–0.58658	–1.12891	–1.07312	–1.10197	–1.0451
	2.0	0.96	–0.5	0.0	–1.28543	–1.57697	–1.34445	–1.10061	–0.71832	–0.5504
				1.0	–0.71406	–1.00117	–1.60621	–1.54291	–1.04879	–0.9841
			0.0	0.0	–1.06669	–1.31824	–1.51492	–1.43133	–0.94568	–0.7482
				1.0	–0.54300	–0.812412	–1.62305	–1.57136	–1.06820	–1.0076
			0.5	0.0	–0.85565	–1.11143	–1.55400	–1.48968	–0.99071	–0.8531
				1.0	–0.2838	–0.6270	–1.67020	–1.59539	–1.07735	–1.0302
0.2	1.0	0.96	–0.5	0.0	–1.46252	–1.65523	–0.54692	–0.51197	–0.52572	–0.4920
				1.0	–0.8063	–1.08956	–1.04631	–0.97241	–1.01927	–0.9435
			0.0	0.0	–1.11487	–1.36935	–0.94555	–0.73352	–0.91627	–0.7043
				1.0	–0.52679	–0.83369	–1.09242	–1.02947	–1.06580	–1.0023
			0.5	0.0	–0.81905	–1.09900	–1.04092	–0.96960	–1.01482	–0.9416
				1.0	–0.21751	–0.56731	–1.13417	–1.07116	–1.10635	–1.0429

decrease the concentration profile significantly in the case of cooling of the fluid. This is because of the fact that concentration gradient accelerates the dispersion of the species.

The values of skin friction parameter, Nusselt number and Sherwood number for various values of physical parameters are recorded in Table I for PST case. The skin friction is found to decrease with the increase of non-Newtonian visco-elastic parameter k_1 as well as porosity parameter (k_2) in the absence of buoyancy effects. This result has significance in industrial applications where power expenditure can be reduced in stretching the sheet by increasing the visco-elastic parameter. From the table we observe that the effect of G_r is to decrease the Nusselt number and Sherwood number. This behaviour is even true in the presence of chemical species diffusion ($G_c \neq 0$).

Conclusions

Natural convective flows, heat and mass transfer due to the combined effect of thermal and species diffusion in a visco-elastic fluid (Walters' liquid B) immersed in a saturated porous medium over a stretching sheet have been investigated numerically. The effects of various physical parameters like visco-elastic parameter, porosity parameter, Grashof number, modified Grashof number, Prandtl number and Schmidt number on horizontal velocity, temperature and concentration profiles are analyzed. The specific conclusions derived from our study are summarized as follows:

- The increase of convective current (Grashof number, G_r) leads to the increase of horizontal velocity profile.

- The introduction of chemical species diffusion (modified Grashof number, G_c) leads to the increase of horizontal velocity profile in both the cases of heating and cooling of the fluid. This observation is even true in the presence of porosity parameter but with reduced magnitude.
- The combined effect of increasing the values of Prandtl number, Grashof number and modified Grashof number is to reduce the temperature profile significantly on the boundary sheet.
- The effects of free convection parameters (G_r, $G_c \neq 0$) and Schmidt (Sc) number are to decrease the concentration distribution in the boundary layer.
- Results of the some of the existing work (Abel and Veena, 1998; Siddappa and Abel, 1985) may be deduced as limiting cases from our results.

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